(fatigue data from Tables XLII and XLIII for $10^6$-$10^7$ cycles life), under the following conditions:

$$(\sigma_\theta)_{max} = 0, \quad (\sigma_\theta)_{min} \geq - \sigma_1 \tag{88}$$

Equation (84) and the definition

$$(\sigma_\theta)_{min} = (\sigma_\theta)_{max} - 2(\sigma_\theta)_r$$

require from (85) that

$$(\sigma_\theta)_{min} = - 2/3 \, \sigma_1 \quad . \tag{89}$$

To obtain conditions (87-89) a fluid-support pressure varying between $q_1$ and $p_1$ is to be found. Because the inner unit consists of only one ring in this case, calculations on the computer are not necessary as they are easily performed by hand. The analysis proceeds as follows:

$$(\sigma_\theta)_{max} = p_o \frac{K^2 + 1}{K^2 - 1} - 2p_1 \frac{K^2}{K^2 - 1} = 0 \quad ,$$

$$p_1 = \frac{p_o}{2} \frac{K^2 + 1}{K^2} = 325,000 \text{ psi} \quad , \tag{90}$$

$$(\sigma_\theta)_{min} = - 2q_1 \frac{K^2}{K^2 - 1} = - 2/3 \, \sigma_1 \quad ,$$

$$q_1 = \frac{K^2 - 1}{K^2} \frac{\sigma_1}{3} = 55,500 \text{ psi} \quad . \tag{91}$$

Thus, it is found that the outer unit must withstand an internal pressure varying between 55,500 psi and 325,000 psi.

The computer code, MULTIR, is used for the outer-unit calculations. A 1/2-inch gap is allowed between the units for the fluid-support pressure, i.e., $r_o = 4.50 + 0.50 = 5.00$ in. for the outer unit. The assumed data are

- wall ratio, $K = 4.0$,
- number of rings, $N = 3$,
- ring radii, $r_o = 5.0$ in., $r_1 = 7.95$ in., $r_2 = 12.61$ in., $r_3 = 20.0$ in.,
- support pressures, $p_N = q_N = 0$,
- minimum bore pressure, $q_o = 55,500$ psi,
- fatigue coefficients, $A_n = 2.86$, $B_n = 1.14$.

Different calculations, 1A - 1D, are performed for rings made from materials with various strengths. Results are given in Table XLV. All four calculations give results that satisfy the requirement of maximum bore pressure of $p_o$ = 325,000 psi. The effect of varying the strength of the rings is indicated. Design 1B has the minimum required interference, $\Delta_1$ = 0.0622 in., corresponding to $\frac{\Delta_1}{r_1} = \frac{0.0622}{7.95}$ = 0.00782 in. in.

TABLE XLV. RESULTS OF COMPUTER CODE MULTIR FOR EXAMPLE DESIGN 1[(a)]

| Design | Design Tensile Strength of Rings, $\sigma_1$, psi 1 | | | Results: Maximum Bore Pressure for $10^6$ Cycles Life | Required Interference[(b)], in. | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | | $\Delta_1$ | $\Delta_2$ |
| 1A | 325,000 | 325,000 | 325,000 | 338,337 | 0.0670 | 0.0739 |
| 1B | 350,000 | 325,000 | 300,000 | 332,699 | 0.0622 | 0.0630 |
| 1C | 375,000 | 350,000 | 300,000 | 345,837 | 0.0658 | 0.0578 |
| 1D | 400,000 | 350,000 | 300,000 | 351,251 | 0.0625 | 0.0578 |

(a) Based entirely on the tensile-fatigue criterion.

(b) Interferences required on the radius. $\Delta_1$ required between rings 1 and 2, and $\Delta_2$ required between 2 and 3.

## Example Design 2

In this design the more conservative shear-fatigue-strength criterion is used for the outer (second) ring of the inner unit and for all three rings of the outer unit. The given data are:

### Inner Unit

wall ratio, K = 3,

number of rings, N = 2,

radii, $r_o$ = 3.00, $r_1$ = 5.1960, $r_2$ = 9.00,

tensile strength of ring 1, $\sigma$ = 300,000 psi,

yield strength of ring 2, $\sigma_y$ = 212,500 psi ($\sigma_y = 0.85\,\sigma_u$, $\sigma_u$ = 250,000 psi),

fatigue coefficients,

$A_1$ = 2.86 and $B_1$ = 0. for ring 1,

$A_2$ = 2.55 and $B_2$ = 2.0 for ring 2,

minimum bore pressure, $q_o$ = 0,

support pressures, $p_2$ = 160,000 psi, $q_2$ = 0.

### Outer Unit

wall ratio, K = 4,

radii, $r_o$ = 9.500 in., $r_1$ = 15.07 in., $r_2$ = 23.90 in., $r_3$ = 38.00 in.,

number of rings, N = 3,

yield strength of rings, $\sigma_y$ = 255,000 psi ($\sigma_y = 0.85\,\sigma_u$, $\sigma_u$ = 300,000 psi),

fatigue coefficients of rings, $A_n$ = 2.55, $B_n$ = 2.00,